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# Small-scale and lateral intermittency of oceanic microstructure in the pycnocline

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#### Abstract

The intermittency correction factor  $\mu^c$  for the Obukhov–Corrsin '-5/3' inertial-convective subrange spectral law is estimated using conductivity (temperature) fluctuation measurements conducted within the microstructure patches of oceanic pycnocline in Monterey Bay. The value of  $\mu^c$  was found to be in the range of 0.46–0.51, depending on the accuracy of calculation, which is applicable to stratified, low-Reynolds-number oceanic turbulence. The intermittency factor  $\mu^c_M$  for mesoscale (up to 1 km) lateral variations of scalar dissipation is estimated as  $0.43 \pm 0.02$ . The breakdown mechanisms of small-scale locally isotropic and mesoscale non-isotropic (lateral) turbulent temperature fluctuations in oceanic turbulence are discussed in the light of the above observations.

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### 1. Introduction

Turbulence by nature is intermittent in space and time. Turbulence variables such as the kinetic energy and scalar dissipation rates are characterized by spiky spatial and temporal fluctuations. In stratified and non-stratified oceanic and atmospheric flows, as well as in other geophysical environs turbulent mixing and microstructure are usually highly intermittent in time as well as in vertical and horizontal spatial directions. It is customary to distinguish the so-called internal and external intermittency of small-scale turbulence (Sreenivasan 2004, Lozovatsky et al 2010). External (or mesoscale) intermittency is considered as an oceanographic phenomenon, although it is also frequent in stably stratified atmospheric layers such as nocturnal boundary layer or upper stratosphere. Variability with typical scales of meters vertically and hundreds of meters horizontally are possible due to such mechanisms as breaking of internal waves, sporadic convective and shear instabilities and baroclinic instability at local fronts. The measured vertical and horizontal sizes of turbulent zones in the upper ocean and lakes are subjected to specific statistical regularities. Their probability distributions appear to be approximately

<sup>3</sup> Also Department of Aerospace and Mechanical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA. log-normal, whereas the distances between turbulent regions follow a double-exponential distribution (Lozovatsky *et al* 1993, Pozdinin 2002). Planella *et al* (2011) reported that the distribution of vertical size of microstructure patches in a small lake is log-normal, with the mean and median values of 0.7 and 0.5 m, respectively.

Internal (small-scale) intermittency is caused by fluctuations of the turbulent kinetic energy  $\varepsilon$  and scalar  $\chi$ dissipation rates at the scales of locally isotropic turbulence, particularly at the inertial-convective and viscous subranges. This intermittency is inherent to all turbulent flows at high Reynolds numbers. Oceanic internal (or 'genuine') intermittency is usually observed at scales from about centimeters to a few meters, and it is usually confined to turbulent patches. Fluctuations of the dissipation rate averaged over the specific volume of radius  $r < L_0$ , where  $L_0$ is an external length scale of turbulence, lead to deviations of turbulent spectra from the '-5/3' laws of Kolmogorov and Obukhov–Corrsin, namely

$$E_{u}(\kappa) = 0.5\tilde{\varepsilon}_{r}^{2/3}\kappa^{-5/3+\mu/9} \quad E_{c}(\kappa) = 0.7\tilde{\chi}_{r}\tilde{\varepsilon}_{r}^{-1/3}\kappa^{-5/3+\mu^{c}/9},$$
(1)

where  $\mu$  and  $\mu^{c}$  are the intermittency factors of the velocity and scalar fluctuations (Monin and Yaglom 1975), which correspond to  $\tilde{\varepsilon}_r$  and  $\tilde{\chi}_r$ . Intermittency factors for log-normal probability distributions of  $\tilde{\varepsilon}_r$  and  $\tilde{\chi}_r$  can be expressed via the variances of the natural logarithms of  $\tilde{\varepsilon}_r$  and  $\tilde{\chi}_r$  as follows:

$$\sigma_{\log \varepsilon_r}^2 = A_{\varepsilon} + \mu \log\left(\frac{L_o}{r}\right) \tag{2a}$$

and

$$\sigma_{\log \chi_r}^2 = A_{\chi} + \mu^c \log\left(\frac{L_o}{r}\right), \qquad (2b)$$

where  $A_{\varepsilon}$  and  $A_{\chi}$  are constants that depend on the characteristics of the mean flow. The term  $\log(L_0/r)$ is the natural logarithm of a ratio between an external turbulent scale L<sub>o</sub> and a scale of the dissipation averaging r. The equations in (2) can be used to estimate  $\mu$  and  $\mu^{c}$ , if the cumulative distribution functions  $F_{\varepsilon}(\log \tilde{\varepsilon}_{r})$  and  $F_{\chi}(\log \tilde{\chi}_r)$  are close to the Gaussian distribution. Gurvich and Yaglom (1967) assumed log normality of the distribution functions of  $F_{\varepsilon}(\log \tilde{\varepsilon}_r)$  and  $F_{\chi}(\log \tilde{\chi}_r)$ , nevertheless many researchers have expressed concern on the use of log-normal model because it is mathematically ill-posed (Novikov 1990, Vanyan 1992, Frisch 1995, Davis 1996, Seuront et al 2005). Although multifractal approaches have been recently employed in several studies to explore intermittency of  $\varepsilon$  in the ocean (e.g. Seuront et al 2005, Lozovatsky et al 2010), it has also been shown that  $\varepsilon$  distributions for oceanic measurements can be approximated as log-normal (e.g. Baker and Gibson 1987, Gibson 1991, Wijesekera et al 1993, Gregg et al 1993, Rehmann and Duda 2000, Lozovatsky and Fernando 2002, Lozovatsky et al 2006). Note that not only the distributions of  $\varepsilon$  within turbulent patches but also the distributions of turbulent scales (Planella et al 2011) and turbulent diffusivities (Paka et al 1999, Roget et al 2006) have been successfully fitted to log-normal models. In this study, we focus on log-normal probability distribution of scalar dissipation  $\sigma_{\log \chi_r}^2$  and its intermittency factors  $\mu^c$ , which have received less attention than  $\sigma_{\log \varepsilon_r}^2$  and  $\mu$ .

#### 2. Results

The microstructure data were obtained in 1990 in the eastern Pacific (Monterey Bay) by using the towing instrument 'GRIF' carrying a pressure sensor and two sensors that extended 0.15 m ahead of the body to measure the mean temperature T and fluctuations of conductivity c at frequencies up to 512 Hz (Paka et al 1999). The towing measurements were conducted in the pycnocline at a depth of 21 m below the sea surface with a towing speed  $\sim 2 \,\mathrm{m \, s^{-1}}$  (the conductivity in the area was mainly influenced by temperature rather than salinity); see Korchagin and Lozovatskii (1998) for details. Figure 1 shows the lateral variations of T(x) along the transect, and the average over consecutive 80 m-length segments was used to evaluate the scalar dissipation rate  $\tilde{\chi}_{80}$ . The transect crossed a 10 km-wide warm mesoscale filament (between  $x \sim 14$  and 24 km) characterized by a sharp temperature increase of  $\sim$ 2.5 °C compared to the background waters. This filament and several smaller (km-scale) lateral interleaving features that resulted from the interaction between warmer coastal and



**Figure 1.** Variations of the temperature  $T_{z=21}$  and averaged scalar dissipation  $\tilde{\chi}_{80}$  along a latitudinal transect in Monterey Bay (the towing depth z = 21 m).



**Figure 2.** An example of the conductivity microstructure record in Monterey Bay. The microstructure patch between 4.147 and 4.158 km was used for multi-scaled averaging of the scalar dissipation  $\tilde{\chi}_r$ . The averaging scale *r* varied from 0.05 to 0.4 m belonging to the Kolmogorov's inertial-convective subrange.

colder upwelling waters (Korchagin *et al* 2001) led to the development of several sharp horizontal thermohaline frontal zones. These fronts affected the lateral distribution of  $\tilde{\chi}_{80}$ , producing mesoscale intermittency.

First, we consider the internal or 'genuine' intermittency of the scalar field induced by small-scale turbulence at spatial scales of the inertial-convective subrange, which is evident in relatively short (~10 m long) statistically homogeneous sections (after denoising and detrending the original signal). A characteristic conductivity segment showing an 'active' 11 m-long microstructure patch (between x = 4.147 and 4.158 km) is given in figure 2. The data allow evaluation of the intermittency factor of scalar fields  $\mu^c$  in the ocean by calculating  $\sigma_{\log \chi_r}^2$ .

To obtain  $\sigma_{\log \chi_r}^2$  using formula (2*b*), the averaging of  $\chi$  should be made over the scale *r* of the inertial-convective subrange of temperature (conductivity). The spectral density  $E_T(\kappa)$  pertinent to the patch in figure 2 exhibits a wide convective-inertial subrange for r = 5-100 cm (figure 3). Thus, it is possible to obtain representative  $\tilde{\chi}_r$  averaged over a number of scales *r* within the realm of Kolmogorov's locally isotropic turbulence. The temperature dissipation rate for a given averaging scale *r* is defined using the isotropic formula  $\tilde{\chi}_r = 6D (dT'/dx)^2$  (Monin and Yaglom 1975), where *D* is the molecular diffusivity of temperature and over bar implies averaging over segments of length *r*. The estimates



**Figure 3.** The spectral density of conductivity (temperature) fluctuations corresponding to the segment shown in figure 2. Vertical lines indicate the averaging scales *r* from the (-5/3) subrange, which were used to calculate the averaged  $\tilde{\chi}_r$ .



**Figure 4.** Log-normal plot of  $F(\log \chi_r)$  for two scales of  $\chi$  averaging: r = 5 cm (diamonds) and r = 40 cm (circles). The standard least-squared fits (for complete range of log  $\chi$  variability) are shown by straight lines 1a and 2. Line 1b is the fit of the first curve in a limited range of log  $\chi$  corresponding to the probability function section between  $\pm 2\sigma_{\log\chi}$  to ignore the tails of the distribution. The intermittency factors  $\sigma^2_{\log\chi_r}$  computed for these fits are given in the legend.

of  $\tilde{\chi}_r$  were obtained for r = 5, 8, 10, 15, 20, 25, 30 and 40 cm, which are exemplified in figure 3 in the background of a typical spectral density  $E_T(\kappa)$ . The number of  $\tilde{\chi}_r$  estimates for this particular microstructure patch, ranging from 226 for r = 5 to 28 for r = 40 cm, were used to calculate the empirical distribution functions  $F_{\chi}(\log \tilde{\chi}_r)$ . Two examples of cumulative distribution functions are shown in figure 4 in the normalized probabilistic coordinates  $y = (\log \tilde{\chi}_r - \langle \log \tilde{\chi}_r \rangle) / \sigma_{\log \tilde{\chi}_r}$  versus  $\log \tilde{\chi}_r$ , which present  $F_{\chi}(\log \tilde{\chi}_r)$  as a straight line, if the distribution is log-normal. The intermittency factors  $\sigma^2_{\log \chi_r}$  were estimated using least-square linear fittings of the empirical functions  $F_{\chi}(\log \tilde{\chi}_r)$ .

Examples shown in figure 4 as well as the majority of estimated empirical distributions  $F_{\chi}(\log \tilde{\chi}_r)$  fit the log-normal model quite well. As pointed out by Yamazaki and Lueck (1990), the log-normal distribution can be used to estimate the averaged (and not local) turbulent energy dissipation rate, provided that the averaging scale *r* is much smaller than the



**Figure 5.** The variance  $\sigma_{\log_{\chi}}^2$  of the natural logarithm of scalar dissipation versus the logarithm of inverse averaging scale 1/r. The squares show the estimates resulting from the least-squared log-normal approximations of the cumulative distribution functions of  $\tilde{\chi}_r$  for the total range of  $\tilde{\chi}_r$  variability. The crossed circles are the estimates of  $\sigma_{\log_{\chi}}^2$  obtained between  $\pm 2\sigma_{\log_{\chi}}$  of the normalized distributions of  $\tilde{\chi}_r$ . The straight and dashed lines approximate the two sets of data in the ranges r = 5-40 cm and r = 8-40 cm, respectively. The corresponding estimates of the intermittency factor  $\mu^c$  equal 0.46 and 0.51.

characteristic scale Lo of the domain and larger than the Kolmogorov scale, which ensures statistical homogeneity of averaged data. The same requirement is expected to apply to scalar dissipation. In our calculations, r varied from 5 to 40 cm and  $L_0$  is on the order of several meters (the segments' size is about 10 m), thus satisfying  $r = L_0$ . Estimates of  $\sigma_{\log \chi_r}^2$ derived from  $F_{\chi}(\log \tilde{\chi}_r)$  distributions are representative of the variance given by (2b). It should be noted that the method used for  $\sigma_{\log \chi_r}^2$  calculations is sensitive to the accuracy of log-normal fitting to field data. The estimates of  $\sigma_{\log \chi_r}^2(1/r)$ were obtained via least-square fitting of  $F_{\chi}(\log \tilde{\chi}_r)$  over a complete range of  $\log \chi$  variability and using a limited range of  $\log \chi$  corresponding to the probability function sections confined by  $\pm 2\sigma_{\log \chi}$ , assuming that the tails of distributions are contaminated by signal errors as well as represent noisy sections of the record. The approach is illustrated in figure 4. It was found that the slopes of the two log-normal approximations are slightly different for r = 5-20 cm, but almost the same for larger averaging scales  $r \sim 25-40$  cm.

Two sets of  $\sigma_{\log \chi_r}^2$  estimate are shown in figure 5. The slope of the linear approximation of  $\sigma_{\log \chi_r}^2(1/r)$  for the entire range of  $\tilde{\chi}_r$  variability (squares) gives  $\mu^c = 0.46 \pm$ 0.04 in 95% of the confidence bounds with a coefficient of determination  $R^2 = 0.98$ . The set of the limited range points (circles) is also well approximated by a linear function (2b), giving  $\mu^c = 0.51 \pm 0.07$  ( $R^2 = 0.97$ ) even though  $\sigma_{\log \chi}^2$ clearly deviates from the straight line at r = 5 cm. This deviation is most probably associated with the end of the inertial-convective subrange that was only approximately estimated at this scale. There is also a possibility that the method of 'best' limiting fitting of  $F_{\chi}(\log \tilde{\chi}_r)$  overestimates the lower tail of the distributions; that is why the limited fit was applied only through r = 8-40 cm (dashed line in figure 5). The corresponding intermittency factor for scalar (temperature) fluctuations based on the atmospheric



**Figure 6.** Variance of the log-normal approximations of cumulative distribution functions  $F_{\chi}(\tilde{\chi}_l)$  calculated for the estimates of scalar dissipation  $\tilde{\chi}_l$  at the consecutive segments of a variable length, l = 0.8–960 m, along the 35 km-long transect (see figure 1). The slope of the linear fit can be interpreted according to (2*b*) as the 'mesoscale' intermittency factor  $\mu_{\rm M}^{\rm c} = 0.43$ .

measurements is in the range  $\mu^{c} = 0.4-0.5$  (Monin and Yaglom 1975).

By extending the intermittency analysis of small-scale scalar fluctuations in confined turbulent patches of the pycnocline (r < 40 cm), such as that in figure 2, to the averaging scales of tens and hundreds of meters over the entire distance of a 35 km-long transect, it is possible to evaluate the 'mesoscale' intermittency factor  $\mu_M^c$  using the estimates of  $\sigma_{\log \chi_l}^2$  obtained from the log-normal approximations of  $F_{\chi}(\tilde{\chi}_l)$  at the consecutive segments of the conductivity record of the variable length l = 0.8-960 m. The variation of  $\tilde{\chi}_{80}$ , shown in figure 1 as an example of the intermittency of mesoscale averaged dissipation  $\tilde{\chi}_l$ , exceeded three orders of magnitude, resulting in higher variances  $\sigma^2_{\log\chi_l}$  compared to  $\sigma^2_{\log \chi_r}$  in more homogeneous individual microstructure patches. The variation of  $\sigma_{\log \chi_l}^2(1/l)$  in 0.8 < l < 960 m is shown in figure 6, where the inferred  $\mu_M^c = 0.43$ with the 95% confidence boundaries between 0.41 and 0.45. The closeness or maybe even equivalence of the intermittency factors of 'genuine' quasi-isotropic small-scale turbulence (caused by the internal intermittency of  $\chi$  in microstructure patches) and that of mesoscale inhomogeneous scalar fields (external intermittency) should be treated with caution because 'mesoscale' intermittency cannot be directly addressed using physics of classical Kolmogorov's cascade of locally homogeneous turbulence. The result, however, may imply some similarities between breakdown mechanisms (Kolmogorov 1941) of scalar fluctuations at various scales of oceanic turbulence, which needs to be studied further via theoretical and experimental approaches.

#### 3. Conclusions

High-frequency towing measurements of conductivity in the upper oceanic pycnocline (z = 21 m) of Monterey Bay in the Eastern Pacific allowed analysis of probability distributions of scalar (temperature) dissipation rate in microstructure patches

by calculating  $\tilde{\chi}_r$  at various averaging scales r = 5-40 cmpertaining to the inertial-convective subrange of turbulent spectra. The log-normal model of  $\tilde{\chi}_r$  distribution (Gurvich and Yaglom 1967) was used to estimate the intermittency correction  $\mu^{c}$  to the Obukhov–Corrsin '-5/3' spectral law that is assumed to be governed by the fluctuations of scalar dissipation. The variance  $\sigma_{\log \chi_r}^2$  was estimated at various r using the log-normal approximation of the empirical cumulative probability functions  $F_{\chi}(\log \tilde{\chi}_r)$ , followed by the calculation of  $\mu^{c}$  based on equation (2b). The intermittency factor  $\mu^{c}$  appears to be 0.46–0.51 depending on the accuracy of the approximation. Similar values of  $\mu^{c}$  have been reported for atmospheric turbulence (Monin and Yaglom 1975) and for the intermittency correction  $\mu$  to the '-5/3' Kolmogorov's subrange (Gibson et al 1970, Gibson 1998) of the velocity spectra of atmospheric turbulence over the sea surface. The dependence of intermittency factors on Reynolds number noted by Lozovatskyet al (2010) suggests that relatively larger values of  $\mu = 0.4-0.5$  reported for atmospheric turbulence and by Wijesekera et al (1993) for oceanic velocity microstructure can be relevant to stratified ocean turbulence, which is mostly associated with relatively low Reynolds numbers, whereas high Reynolds number flows are usually specified by smaller values of  $\mu \sim 0.2-0.25$  (e.g. Seuront et al 2005). The scalar intermittency factor  $\mu^{c}$  could be subjected to the same variability as its energy dissipation rate counterpart  $\mu$ .

Mesoscale intermittency of oceanic microstructure in the pycnocline at lateral scales from meters to kilometers is characterized by the intermittency factor  $\mu_M^c = 0.43 \pm$ 0.02, which may imply an essential similarity between the small-scale (locally isotropic) and mesoscale (non-isotropic lateral) scalar-fluctuation breakdown mechanisms in the ocean.

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